4. Equation of a plane which passes through three given points (a_1, b_1, c_1), (a_2, b_2, c_2) and (a_3, b_3, c_3) is

$$\begin{vmatrix} x & y & z & 1 \\ a_1 & b_1 & c_1 & 1 \\ a_2 & b_2 & c_2 & 1 \\ a_3 & b_3 & c_3 & 1 \end{vmatrix} = 0.$$

Problems:- 1. Find the equation of a plane where perpendicular drawn from the origin to the point $(2, 3, -1)$ on the plane & direction ratios of this perpendicular are $3, 1, 4$.

Ans:- Length of the perpendicular from $(0.0.0)$ to $(2,3,-1)$ on the ~~pl~~ plane is $p = \sqrt{(0-2)^2+(0-3)^2+(0+1)^2}$

$= \sqrt{4+9+1} \qquad = \sqrt{14}$.

given $3, 1, 4$ are the direction ratios of this perp.

Let l, m, n be the direction cosines, then

$$l = \frac{3}{\pm\sqrt{3^2+1^2+4^2}} = \frac{3}{\pm\sqrt{26}} \; ; \; m = \frac{1}{\pm\sqrt{26}} \; ; \; n = \frac{4}{\pm\sqrt{26}}.$$

Hence the equation of the plane is

$$\frac{3}{\pm\sqrt{26}} x + \frac{1}{\pm\sqrt{26}} y + \frac{4}{\pm\sqrt{26}} z = \sqrt{14}$$

or, $3x + y + 4z = \pm\sqrt{14}\sqrt{26}$

or, $3x + y + 4z = \pm\sqrt{364}$ Ans.

Q: Find the equation of the plane which passes through the point $(2,-1,3)$, $(1,4,-2)$ & $(3,1,0)$.

A:- Equation of the plane which passes through the points $(2,-1,3)$, $(1,4,-2)$ & $(3,1,0)$ is

$$\begin{vmatrix} x & y & z & 1 \\ 2 & -1 & 3 & 1 \\ 1 & 4 & -2 & 1 \\ 3 & 1 & 0 & 1 \end{vmatrix} = 0$$

$$\text{or, } x\begin{vmatrix} -1 & 3 & 1 \\ 4 & -2 & 1 \\ 1 & 0 & 1 \end{vmatrix} - y\begin{vmatrix} 2 & 3 & 1 \\ 1 & -2 & 1 \\ 3 & 0 & 1 \end{vmatrix} + z\begin{vmatrix} 2 & -1 & 1 \\ 1 & 4 & 1 \\ 3 & 1 & 1 \end{vmatrix} - 1\cdot\begin{vmatrix} 2 & -1 & 3 \\ 1 & 4 & -2 \\ 3 & 1 & 0 \end{vmatrix} = 0$$

$$\text{or, } x\{-1(-2-0)-3(4-1)+1(0+2)\} - y\{2(-2-0)-3(1-3)+1(0+6)\} + z\{2(4-1)+1(1-3)+1(1-12)\} - 1\{2(0+2)+1(0+6)+3(1-12)\} = 0$$

$$\text{or, } x(2-9+2) - y(-4+6+6) + z(6-2-11) - 1(4+6-33) = 0$$

$$\text{or, } -5x - 8y - 7z + 23 = 0$$

$$\text{or. } 5x + 8y + 7z - 23 = 0 \quad \text{required equation.}$$

Condition of Parallelism & perpendicularity:

Let the two equations of planes be $a_1x+b_1y+c_1z+d_1=0$ & $a_2x+b_2y+c_2z+d_2=0$

These two planes will be Parallel if $\frac{a_1}{a_2} = \frac{b_1}{c_2} = \frac{c_1}{c_2}$

& these two planes will be perpendicular to each other if $a_1a_2+b_1b_2+c_1c_2=0$

The Plane

1. General equation of a plane in 3 dimensional space is $ax+by+cz+d=0$

If this plane passes through the origin $(0,0,0)$, then

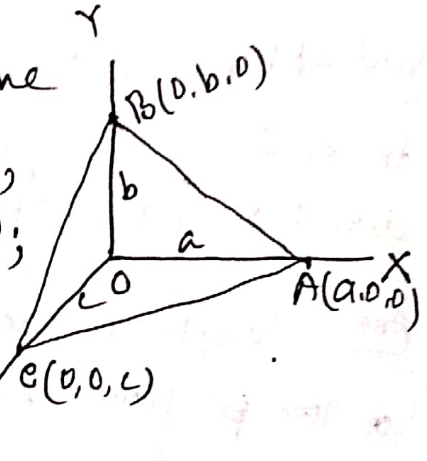
$$a\cdot 0+b\cdot 0+c\cdot 0+d=0$$

$$\text{or, } d=0$$

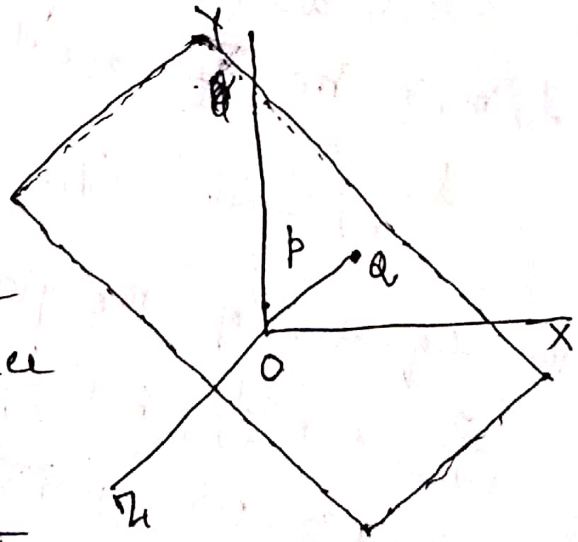
$\therefore$ Equation of the plane passing through the origin is

$$ax+by+cz=0$$

2. Intercept form: Equation of a plane which intersects the X-axis at $A(a,0,0)$, Y axis at $B(0,b,0)$ & Z axis at $C(0,0,c)$; is $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$, where a, b, c are the intercepts from X, Y, Z axes respectively.



3. Normal Form:



Normal form of the equation of a plane in 3-dimensional space is $lx+my+nz=p$, where l, m, n are the direction cosines of the perpendicular drawn from the origin to the plane and p is the length of that perpendicular.

Q: Find the equation of a plane which passes through the intersection of the planes $2x+3y+5z+1=0$ & $x-2y+7z+4=0$ and (i) passes through the point $(1,-2,0)$

(ii) Parallel to the plane $5x+y+2z+1=0$,

(iii) Perpendicular to the plane $3x-y+4z+2=0$.

A:- Equation of a plane which passes through the intersection of the planes $2x+3y+5z+1=0$ & $x-2y+7z+4=0$ is

$$2x+3y+5z+1+k(x-2y+7z+4)=0 \quad \text{—— (1)}$$, where k is constant.

(i) Since the plane (1) passes through the point $(1,-2,0)$,

$\therefore\ 2\cdot1+3(-2)+5\cdot0+1+k(1-2\cdot-2+7\cdot0+4)=0$

$\cancel{-4+1+k}\quad -3+k(9)=0$

$\therefore\ k=\frac{1}{3}$

From (1); $2x+3y+5z+1+\frac{1}{3}(x-2y+7z+4)=0$

$6x+9y+15z+3+x-2y+7z+4=0$

or, $7x+7y+22z+7=0$ which is the required eqn.

(ii) From (1); $2x+3y+5z+1+k(x-2y+7z+4)=0$

or, $(2+k)x+(3-2k)y+(5+7k)z+1+4k=0$ —— (2)

Since this plane (2) is parallel to the plane $5x+y+2z+1=0$

$\therefore\ (2+k)\cdot5+(3-2k)\cdot1+(5+7k)\cdot2=0$

or, $10+5k+3-2k+10+14k=0$

or, $17k+23=0$

$\therefore\ k=-23/17$

From (2), $2x+3y+5z+1-\frac{23}{17}(x-2y+7z+4)=0$

or, $34x+51y+85z+17-23x+46y-161z-92=0$

or, $11x+97y-76z-75=0$; required equation

(ii) From equation ①; $2x+3y+5z+1+k(x-2y+7z+4)=0$ ——— ②

or, $(2+k)x+(3-2k)y+(5+7k)z+1+4k=0$

Since the plane ② is parallel to the plane $5x+y+2z+1=0$

$\therefore \dfrac{2+k}{5} = \dfrac{3-2k}{1} = \dfrac{5+7k}{2}$

Taking $\dfrac{2+k}{5} = \dfrac{3-2k}{1}$

$15-10k = 2+k$

$-11k = -13$

$k = 13/11$

taking $\dfrac{3-2k}{1} = \dfrac{5+7k}{2}$

$5+7k = 6-4k$

$11k = -1$

$k = -1/11$

taking $\dfrac{2+k}{5} = \dfrac{5+7k}{2}$

$25+35k = 4+2k$

$33k = -21$

$k = -21/33$

putting $k=\frac{13}{11}$ in ②, we get the required eqn. of the plane

$2x+3y+5z+1+\frac{13}{11}(x-2y+7z+4)=0$

$22x+33y+55z+11+13x-26y+91z+52=0$

$35x+7y+146z+63=0$ Ans.

Similarly by the other values of $k = -\frac{1}{11}$ & $-\frac{21}{33}$, we can get other two equations.

(iii) From equation ①, $2x+3y+5z+1+k(x-2y+7z+4)=0$ ——— ③

or, $(2+k)x+(3-2k)y+(5+7k)z+1+4k=0$

Since the plane ③ is perpendicular to the plane $3x-y+4z+2=0$

$\therefore (2+k)\cdot 3+(3-2k)\cdot(-1)+(5+7k)\cdot 4=0$

or, $6+3k-3+2k+20+28k=0$

or, $33k+23=0$

$\therefore k=-23/33$

Hence from ③, $2x+3y+5z+1-\frac{23}{33}(x-2y+7z+4)=0$

$66x+99y+165z+33-23x+46y-161z-92=0$

or, $33x+145y+4z-59=0$ which is the required equation.